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EVALUATION OF PRECISION AND ACCURACY IN THE CALIBRATION OF HYDROPHONES

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Evaluation of Precision and Accuracy in the Calibration of Hydrophones

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13. ABSTRACT

The Underwater Sound Reference Division has made a preliminary study of the accuracy or bias in measurement of hydrophone sensitivity by the comparison method and of the precision or repeatability of the measurements. To evaluate the various calibration systems, two standard hydrophones were calibrated a number of times by reciprocity with a minimum of electronic equipment. The averages from the repetitions were taken as "true" values of hydrophone sensitivity. Three sets of calibration data then were obtained for both hydrophones with each calibration system, and the results were compared with the "true" values to obtain estimates of the bias and confidence interval associated with each system. The comparisons showed that the biases generally are less than ±0.3 dB with 95% certainty and that a confidence interval of ±0.6 dB generally will contain the "true" values with 95% certainty. None of the data on systems that were operating properly fell outside a range of -1.0 to +0.4 dB about the "true" values.

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Calibration facilities						
Low frequency calibrations						
Calibration methods--test facilities						
Reciprocity calibrations						

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EVALUATION OF PRECISION AND ACCURACY IN THE CALIBRATION OF HYDROPHONES

INTRODUCTION

When a standardizing institution such as the National Bureau of Standards issues a certificate for a standard, it attributes to that standard a numerical value together with a statement about the uncertainty of this value [1]. An uncertainty statement is necessary in calibration because from a philosophical viewpoint "absolute certainty is a privilege of uneducated minds and fanatics" [2]. The standardizing laboratory never can be absolutely certain that the certified value falls within the limits stated in the certificate; however, from a practical viewpoint, it is quite justified in having confidence in a calibration value based on its knowledge of the calibration process and the physical nature of the device calibrated. This confidence generally is transmitted in the form of an uncertainty statement.

The primary function of the Underwater Sound Reference Division (USRD) of the Naval Research Laboratory (NRL) is to serve as the Navy's standardizing institution for underwater electroacoustic measurements. Because stable electroacoustic devices are required if calibration is to remain valid for a reasonable length of time and under a range of environmental conditions, and because a thorough physical knowledge of the standards is necessary to good calibration, an important role in fulfilling the primary mission is played by the Standards Branch of the USRD in the development and manufacture of stable, transportable, standard underwater electroacoustic transducers for Navy activities and contractors.

At the USRD, knowledge of the characteristics of underwater electroacoustic transducers and of the calibration process has led to the belief that the common calibration procedures employed have negligible bias and that the accuracy associated with these measurements is ±1 dB. No confidence limits are specified, and the accuracy is an educated guess based on experience. The purposes of this study are: (1) to establish valid confidence limits on the calibration outputs of the existing calibration facilities; (2) where possible, separate and identify systematic and random errors associated with each facility; and (3) where appropriate, recommend changes in calibration procedures, or equipment, or both, to decrease the confidence interval associated with the calibration measurements.

Because the accumulation of enough data to establish valid confidence limits is a time-consuming process, and the USRD calibration systems' workloads are such that time is available to acquire the needed data only infrequently, the results in this report are based on a very limited amount of data; they should be viewed as only an initial look at a problem that will require a more thorough system-by-system analysis. Results of this study do indicate which systems require the most immediate attention and which can continue to be used with a reasonable degree of certainty.

As measures of experimental error, precision and accuracy as defined by Mandel [1] are used. Precision is defined in terms of its opposite, imprecision. Imprecision is the amount of scatter exhibited by results obtained from repeated experimental measurements. There will always be some imprecision associated with experimental measurements because of lack of fineness of scale on some measuring devices, although the results may seem to be precise. The accuracy of an experimental measurement can be defined in terms of the bias or systematic error of the experimental measurements or as the difference between the true value and the expected value, which can only be estimated from repeated measurements. From these definitions, one can see that a measurement procedure can be considered precise but not accurate or accurate but not precise. Using these definitions avoids some of the philosophical problems associated with another common definition of accuracy (that is concerned with both bias and precision) and is admittedly taking the easy way out [3]. In brief, we avoid the situation in which a procedure having a small bias and a high precision is called more accurate than an unbiased procedure of low precision. Uncoupling these terms allows one to distinguish between procedures in which repetition can reduce the confidence interval about an unbiased mean and those in which repetition beyond a certain limit does no good because the mean is biased.

The first problem one encounters in establishing confidence intervals for the calibration of hydrophones is that of arriving at a philosophically acceptable definition of the true value of hydrophone sensitivity in terms of ideal deterministic parameters, because the true value must be known before one can say how much error is involved in a single measurement. This problem has been solved to some extent by the application of electroacoustic two-port theory in reciprocity calibration [4]. Under ideal conditions, reciprocity calibration is supposed to be an absolute method; however, several conditions must be satisfied to justify the conclusion that a free-field reciprocity calibration results in an unbiased or absolute value of hydrophone sensitivity. These conditions are the existence of a free sound field, sufficient space to permit measurements in the far field of the source, linearity, and infinite signal-to-noise ratio (SNR), none of which can be achieved in practice, so some bias or inaccuracy in the measurement is inevitable. This paper discusses the biases to be expected in limited-water reciprocity calibration and attempts to estimate their magnitude or minimize their effects. Hydrophones calibrated repeatedly after minimizing biases have been used in this study as primary standards having "known" calibrations to provide an

initial evaluation of the practices and systems used at the USRD in the established calibration facilities.

Early in this investigation it was recognized that the evaluation of system accuracy and precision does not provide a confidence interval and bias statement for every type of hydrophone that might be calibrated on that particular system; however, some general statements about extreme confidence intervals and bias may be made for certain categories of hydrophones such as the NRL-USRD standards operated below resonance for a particular system. This region for these standards is of primary interest to the NRL-USRD in fulfilling its primary mission of serving as the standardizing institution for underwater electroacoustic measurements. The accuracy and precision, which in our other measurements is of secondary interest, depend on the instrument upon which the measurements are being performed, and often is so dependent upon the instrument that additional charges sometimes should be made to the customer if very good evaluation of the measurements is desired. The present investigation is confined to the problem of specifying accuracy and precision of standard hydrophones at frequencies below resonance.

THEORY

To evaluate the precision and accuracy in calibration of hydrophones by the reciprocity method requires an understanding of electroacoustic reciprocity and calibration practice. MacLean [4] developed the reciprocity method for the absolute calibration of electroacoustic transducers. His development was confined to a lumped-parameter treatment, which is not generally valid when the motion of the electroacoustic transducer is not the same for projecting as it is for receiving sound waves. Foldy and Primakoff [5] proved that the reciprocity method was independent of the particular characteristics of the transducer so that the differences in motion in projecting and receiving do not matter. Their rather general proof was based on generalized impedances, Green's functions, and both acoustic and electroacoustic reciprocity. Bobber [6] has discussed both the electroacoustic reciprocity calibration method and practice thoroughly in his recent book on underwater electroacoustic measurements. Also, an understanding of the effect of resonances and radiation impedance on hydrophone sensitivity is essential in evaluating the precision and accuracy of calibration, because a properly designed hydrophone can be calibrated with better accuracy than can a poorly designed one.

Here, the basic definition of hydrophone sensitivity will be reviewed using lumped-parameter, electromechanical two-port theory instead of the more generalized approach of Foldy and Primakoff. Then bias in the measurement of hydrophone sensitivity by the reciprocity method will be discussed. This will be followed by a review of the statistical concepts associated with the acquisition and averaging of the hydrophone sensitivity estimates.

Electromechanical Two-Port Theory for a Hydrophone

If a transducer is linear and reciprocal, it can be represented as a simple electromechanical two-port device as illustrated in Fig. 1. The

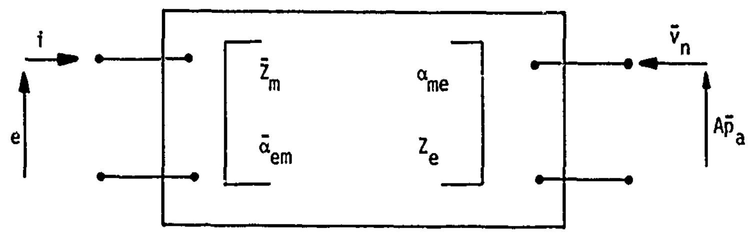


Fig. 1. Representation of an electromechanical two-port device.

two-port equations for this device (assuming the vibratory mode is the same whether projecting or receiving) are given by

$$A\bar{p}_a = \bar{Z}_m \bar{v}_n + \alpha_{me} i \tag{1}$$

and

$$e = \bar{\alpha}_{em} \bar{v}_n + Z_e i, \tag{2}$$

where $\bar{p}_a$ is the average applied pressure over the area A of the mechanical port, $\bar{v}_n$ is the average velocity normal to A, i is the electrical current in the electromechanical two port, e is the voltage across the electrical port, and the impedance parameters are defined by the equations

$$\bar{Z}_m = \left.\frac{A\bar{p}_a}{\bar{v}_n}\right|_{i=0}, \tag{3}$$

$$\alpha_{me} = \left.\frac{A\bar{p}_a}{i}\right|_{\bar{v}_n=0}, \tag{4}$$

$$\bar{\alpha}_{em} = \left.\frac{e}{\bar{v}_n}\right|_{i=0}, \tag{5}$$

and

$$Z_e = \left.\frac{e}{i}\right|_{\bar{v}_n=0}. \tag{6}$$

The quotient of open-circuit voltage divided by the force on area A is obtained from Eqs. (1) and (2) as

$$e_{oc}/A\bar{p}_a = \bar{\alpha}_{em}/\bar{Z}_m. \tag{7}$$

To obtain an output voltage from an electromechanical device, a force must be delivered to the mechanical port. Force is delivered to a hydrophone through vibration of the medium. The mechanical port looks out upon the medium and its boundaries as a mechanical radiation impedance $\bar{Z}_{mr}$. The coupling of the force from the mechanical radiation impedance to the mechanical port probably can be best understood in terms of a purely mechanical system. From the dual mechanical port of Fig. 1, one obtains Fig. 2; force Ap is transmitted by the rod-connected system from $\bar{Z}_{mr}$ to $\bar{Z}_m$ when the electrical port is open-circuited. Note that the rules applicable to a mechanical system are not the same as those for an electrical circuit. Impedances in parallel add because they are defined as the through variable (force) divided by the across variable (velocity). Using the mechanical analog of Norton's equivalent circuit, one obtains the mechanical system shown in Fig. 3, where $\bar{F}_b$ is the average force that

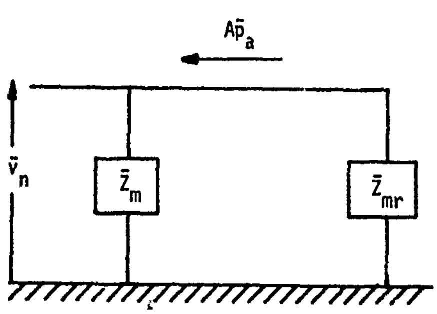


Fig. 2. Mechanical system for an open-circuited electromechanical transducer.

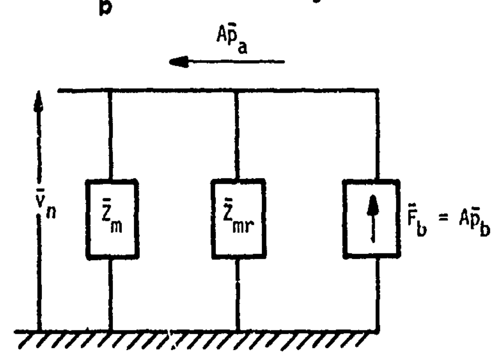


Fig. 3. Equivalent mechanical system using the analog of Norton's equivalent circuit.

arises when the diaphragm of the transducer is blocked. The blocked force is equal to the product of the area and the average blocked pressure $\bar{p}_b$. From Fig. 3 it is deduced that

$$\frac{\bar{p}_a}{\bar{p}_b} = \frac{\bar{Z}_m}{\bar{Z}_m + \bar{Z}_{mr}}. \tag{8}$$

The average blocked pressure is related to the free-field pressure p_{ff} by the equation

$$\bar{p}_b = Dp_{ff} \tag{9}$$

where D is the diffraction constant of the transducer. The receiving sensitivity of the transducer is obtained from Eqs. (7), (8), and (9):

$$M \equiv e_{oc}/p_{ff} = \bar{\alpha}_{em} AD(\bar{Z}_m + \bar{Z}_{mr})^{-1}. \tag{10}$$

Bias in Reciprocity Measurement of Hydrophone Sensitivity

The reciprocity method calibration equation for free-field voltage sensitivity given by Bobber [6] in terms of deterministic quantities or true values is

$$M = \left(\frac{e_{TH} e_{PH}}{e_{PT} i_T} \cdot \frac{2d \times 10^{-17}}{\rho f}\right)^{1/2}, \tag{11}$$

where P, T, and H are symbols for the projector, transducer, and hydrophone, respectively; the e's are open-circuit voltages under noise- and interference-free conditions, and the first subscript refers to the sound emitter while the second refers to the receiver; i_T is the transducer current; d is the measurement distance; ρ is the water density; and f is the frequency. Equation (11) gives the sensitivity in volts per micropascal if the e's are in volts, i_T in amperes, d in centimeters, ρ in grams per cubic centimeter, and f in hertz. In measuring the quantities e_{TH}, e_{PH}, and e_{PT}, the distance d is measured each time so that each measured e has a voltage measurement error and a distance measurement error associated with it. In free-field, far-field measurements, the functional dependence of the true values of the open-circuit voltages is given by

$$e = e_0/d, \tag{12}$$

where e_0 is the voltage that would appear at distance d_0. Because measurements are made in the presence of noise, an estimate of e for distance d is given by

$$\hat{e} \approx \widehat{(e^2 + \sigma_n{}^2)}^{1/2}(d/\hat{d}) \tag{13}$$

where the circumflex represents an estimated quantity for a random variable, and σ_n is the root-mean-square (rms) value of an assumed zero-mean, stationary, Gaussian noise, and e is an rms value. Here it was assumed also that signal-to-noise ratio does not change with small changes in distance. An estimate of M can now be written as

$$\hat{M} \simeq \left[\frac{\widehat{(e_{TH}^2 + \sigma_{Hn}^2)}^{1/2} \; \widehat{(e_{PH}^2 + \sigma_{Hn}^2)}^{1/2} \; \hat{d}_{PT}(2d^2 \times 10^{-17})}{\widehat{(e_{PT}^2 + \sigma_{Tn}^2)}^{1/2} \; \hat{i}_T \hat{d}_{TH} \hat{d}_{PH} \hat{\rho} \hat{f}} \right]^{1/2} \tag{14}$$

In Eq. (14) all of the right-hand quantities with the circumflex can be considered independent. The question of distribution and bias in the estima.. n of M now arises. Provided that the errors are small in comparison with the quantities being measured, the approximate error in computing the sensitivity can be found by taking the variation of Eq. (14). Doing this, dividing the result by Eq. (14), and replacing the estimated quantities in Eq. (14) by their deterministic counterparts, one obtains

$$\frac{\delta M}{M} \simeq \frac{1}{2} \left(\frac{\delta e_{TH}}{e_{TH}} + \frac{\delta e_{PH}}{e_{PH}} + \frac{\delta d_{PT}}{d} - \frac{\delta e_{PT}}{e_{PT}} - \frac{\delta i_T}{i_T} - \frac{\delta d_{TH}}{d} - \frac{\delta d_{PH}}{d} - \frac{\delta \rho}{\rho} - \frac{\delta f}{f} \right) \tag{15}$$

for the relative uncertainty in the derived measurement of hydrophone sensitivity by the reciprocity method. For a measurement process, relative uncertainty is a random variable. To compute a confidence interval for the measurement process, it is necessary to know or to approximate the probability density function for the relative uncertainty of the process. An uncertainty is associated also with the measurement of the probability density function for the relative uncertainty. For these reasons, relative uncertainty often is considered to be a zero-mean, Gaussian variable, but without adequate justification. For the case of free-field measurements made with freely suspended electroacoustic transducers, one can argue that the assumption is valid under certain conditions. For this case, the only factors contributing significantly to the relative uncertainty of hydrophone sensitivity are the relative uncertainties of voltage and distance values. The voltage estimates can be assumed to arise from a Gaussian process if noise is stationary and if the time constant of the measuring instrument is large in comparison with the reciprocal of the bandwidth of the interfering noise because of the central limit theorem. Distance can be measured between the centers of the riggings for the projectors and hydrophones, but the distance between the acoustic centers of these instruments may vary randomly and may also contain a bias distance. If these instruments are cylindrical and can be suspended by their cables, one may argue that the distance can be measured at the air-water interface and that distance bias can be ignored. Repeated rigging then probably would result in the relative uncertainty of distance being an unbiased Gaussian variable. Because the sum or difference of Gaussian variables results in a Gaussian variable, the relative uncertainty of hydrophone sensitivity as well as the estimates of hydrophone sensitivity can be considered a biased Gaussian variable. Hydrophone sensitivity is commonly expressed on a logarithmic basis referenced to 1 μbar or to 1 μPa. It is well known that the average of the logs always will be less than or equal to the log of the average. Hershey

[7] has shown that when the ratio of the standard deviation to the mean σ/μ is less than 1/10 (fluctuation $\simeq \pm 1$ dB), then there is less than a 0.05-dB difference between the average of the logs and the log of the average for a Gaussian process when the average is taken over fewer than 30 values.

Careful attention to the output voltage of transducer or hydrophone will ensure that the readings are as free of bias as possible. There are four apparent sources of bias in the voltage readings: (1) ambient noise, (2) crosstalk, (3) multipath transmission, and (4) bias in reading and in voltmeter calibration. The effect of ambient noise is as indicated in Eq. (13). Keeping the signal-to-noise ratio above 20 dB reduces noise bias to less than 0.05 dB on the average. Crosstalk can be a more serious problem and should be kept 40 dB below signal level to limit bias to less than 0.05 dB. Bias in voltmeter calibration and reading is virtually eliminated by choice of a good voltmeter. Effects of multipath transmission caused by lack of true free-field conditions must be taken into account.

When the free-field sensitivity of a hydrophone is measured in a bounded environment where multipath transmission occurs, a change of sensitivity arises from the change in mechanical radiation impedance and in bias caused by multipath acoustic pressures. As an example of how a boundary affects radiation impedance, consider a small spherical transducer P of radius a operated near an air-water interface as illustrated

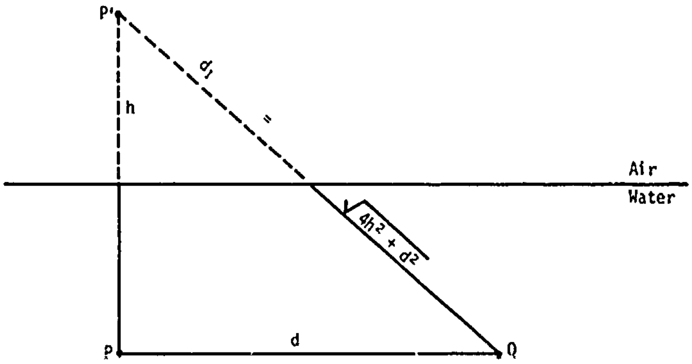


Fig. 4. Geometry for projector P, which establishes pressure P_0 in the vicinity of an air-water interface.

in Fig. 4. Assume h >> a and the wavelength λ radiated is large in comparison with a (monopole source). Let the spherical radiator pulsate with radial velocity

$$v_r = v_{r0} e^{j\omega t}, \tag{16}$$

where ω is the angular frequency, and v_{r0} is the radial velocity at the

surface of the sphere. When the amplitude of the vibrations is small in comparison with a, a << d. Under free-field conditions, the acoustic pressure at point Q, distance d from the center of the sphere, is given by

$$p_{ff}(d,t) = \frac{p_s a}{d} e^{j[\omega t - k(d-a)]}, \tag{17}$$

where p_s is the pressure amplitude at the surface of the sphere and k is the wave number. The specific acoustic radiation impedance is obtained as

$$(Z_{sr})_{ff} \equiv p_{ff}(a,t)/v_r = p_s/v_{r0} \tag{18}$$

for the free-field case. Because the wavelength is assumed to be large in comparison with the radius of the sphere, the specific acoustic radiation impedance can be shown to be

$$(Z_{sr})_{ff} = \frac{\rho c k^2 a^2 + j\rho c k a}{1 + k^2 a^2}, \tag{19}$$

where c is the speed of sound in water and ρ is the mean density. Now consider the same spherical radiator pulsating with the same radial velocity but located at depth h below the air-water interface. The acoustic pressure at a point d distant from the center of the sphere and at the same depth is given by

$$p_{aw}(d,t) = p_s a\left(\frac{e^{-jk(d-a)}}{d} - \frac{e^{-jk(d_1-a)}}{d_1}\right)e^{j\omega t}, \tag{20}$$

where the subscript aw refers to the air-water interface, and

$$d_1 = (4h^2 + d^2)^{\frac{1}{2}}. \tag{21}$$

The specific acoustic radiation impedance for this case is given by

$$(Z_{sr})_{aw} \equiv \frac{p_{aw}(a,t)}{v_r} = (Z_{sr})_{ff}\left(1 - \frac{e^{-2jkh}}{2kh}\right). \tag{22}$$

Specific acoustic impedances are converted to mechanical impedances by multiplying by the area of the sphere. The mechanical radiation impedances are approximately

$$(Z_{mr})_{ff} \simeq \frac{4\pi a^2(\rho\omega k a^2 + j\omega\rho a)}{1 + k^2 a^2} \tag{23}$$

and

$$(Z_{mr})_{aw} \simeq (Z_{mr})_{ff}\left(1 - \frac{e^{-2jkh}}{2kh}\right). \tag{24}$$

The effect of the mechanical radiation impedance in the presence of an air-water interface on hydrophone sensitivity is evident in the universal electrical equivalent circuit of a Class I transducer (Fig. 5), which is given

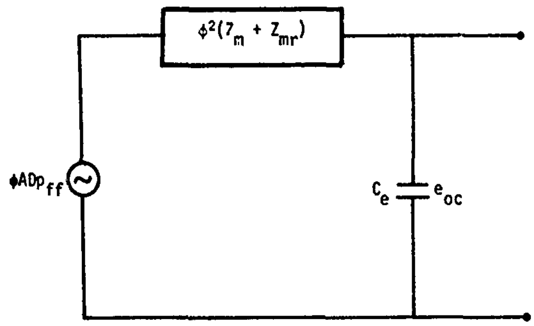


Fig. 5. Electrical equivalent circuit for open-circuited Class I transducer.

by Fischer [8] (some notational changes have been made). With the circuit viewed as a voltage divider,

$$M \equiv \frac{e_{oc}}{p_{ff}} = \frac{\phi AD}{1 + j\omega\phi^2 C_e(Z_m + Z_{mr})} \tag{25}$$

is obtained for hydrophone sensitivity, where ϕ is the electromechanical conversion factor. The effect of radiation impedance on sensitivity is negligible far below resonance, where sensitivity is controlled primarily by the mechanical compliance C_m of the transducer element and is essentially constant with frequency provided that D is constant with frequency over this region. Sensitivity is most seriously affected by interfering boundaries at resonance because the radiation mass can become comparable with the vibrating mass of the transducer element and radiation reactance is zero. For example, the vibrating mass of a spherical, rigidly backed ceramic shell vibrating in the pulsating mode is about half that of the shell, while the radiation mass is three times that of the displaced water (for the free-field case). For other hydrophone configurations, the effect of water mass loading on sensitivity decreases with increasing directivity.

An example of another source of bias in the calibration of hydrophones by reciprocity is that produced by the interference pressure at an air-water interface (excluding all other effects). The signal reflected from the interface and the direct signal sum constructively and destructively to produce a cyclic amplitude as a function of frequency, which is known as the interference cycle because it is periodic with frequency. When the estimates of hydrophone sensitivity are made over an interference cycle,

the bias B(M) relative to free-field calibration is

$$\frac{B(M)}{M} = \frac{M_{aw}}{M} - 1, \tag{26}$$

where M_{aw} denotes the sensitivity measured near the air-water interface. The reciprocity equation gives

$$\frac{B(M)}{M} = \left[\frac{(e_{PH})_{aw}\ (e_{TH})_{aw}\ e_{PT}}{e_{PH}e_{TH}\ (e_{PT})_{aw}}\right]^{\frac{1}{2}} - 1. \tag{27}$$

If it is assumed that far below hydrophone resonance the sensitivities for free-field and near the air-water interface are the same, then

$$(e_{PH})_{aw} \simeq M(p_H)_{aw}, \tag{28}$$

and

$$e_{PH} \simeq Mp_H, \tag{29}$$

etc., where $(p_H)_{aw}$ and p_H are the pressures at the hydrophone near an air-water interface and in a free field, respectively. Application of these relationships to Eq. (26) gives

$$\frac{B(M)}{M} = \left[\frac{(e_{TH})_{aw}}{e_{TH}}\right]^{\frac{1}{2}} - 1. \tag{30}$$

The actual bias obtained will depend upon the detection method used in obtaining e_{TH} and $(e_{TH})_{aw}$.

For rms detection in a free field, let the instantaneous pressure field at the hydrophone due to the reciprocal transducer be

$$p_H(d,t) \propto \frac{\sin(\omega t - kd)}{d}. \tag{31}$$

After squaring, time averaging, and taking the square root of this expression, one obtains

$$p_H \propto 1/d\sqrt{2}. \tag{32}$$

In the presence of an air-water interface, the pressure at the hydrophone is

$$[p_H(d,t)]_{aw} \propto \frac{\sin(\omega t - kd)}{d} - \frac{\sin[\omega t - k(4h^2 + d^2)^{1/2}]}{(4h^2 + d^2)^{1/2}}. \tag{33}$$

Squaring, time averaging over an integral number of periods, and taking the square root of this equation gives

$$\left(\int_0^T [p_H(d,t)]^2_{aw} dt\right)^{1/2} \propto \left(\frac{1}{2d^2} + \frac{1}{2(4h^2 + d^2)} - \frac{\cos k[(4h^2 + d^2)^{1/2} - d]}{d(4h^2 + d^2)^{1/2}}\right)^{1/2}. \tag{34}$$

Averaging Eq. (34) over an interference cycle produces

$$(p_H)_{aw} \propto \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{8h^2 + 4d^2}{4d^2(4h^2 + d^2)} - \frac{\cos k\Delta x}{d(4h^2 + d^2)^{1/2}}\right)^{1/2} d(k\Delta x), \tag{35}$$

where $\Delta x \equiv (4h^2 + d^2)^{1/2} - d$.
The ratio of the rms-detected pressures averaged over an interference cycle is then

$$\frac{(p_H)_{aw}}{p_H} = \frac{1}{2\pi}\left(\frac{4h^2 + 2d^2}{4h^2 + d^2}\right)^{1/2} \int_0^{2\pi}\left(1 - \frac{2d(4h^2 + d^2)^{1/2}}{4h^2 + 2d^2}\cos k\Delta x\right)^{1/2} d(k\Delta x). \tag{36}$$

Let $b = d/h$. Then Eq. (36) becomes

$$\frac{(p_H)_{aw}}{p_H} = \frac{1}{2\pi}\left(\frac{4 + 2b^2}{4 + b^2}\right)^{1/2} \int_0^{2\pi}\left(1 - \frac{2b(4 + b^2)^{1/2}}{4 + 2b^2}\cos k\Delta x\right)^{1/2} d(k\Delta x). \tag{37}$$

Expanding the integral into a binomial series and integrating, one obtains

$$\frac{(p_H)_{aw}}{p_H} =$$

$$\left(\frac{4 + 2b^2}{4 + b^2}\right)^{1/2} \left\{1 - \sum_{n=1}^{\infty} \frac{[1\cdot 5 \cdots (4n - 3)][1\cdot 3 \cdots (2n - 1)]}{(2n)!\ [2\cdot 4 \cdots (2n)]} \left(\frac{b^2(4 + b^2)}{(4 + 2b^2)^2}\right)^n\right\} \tag{38}$$

for all values of b.

Equations (30) and (38) have been used to determine the bias in calibrating a small ($a \ll \lambda$, $h \gg a$) nonresonant hydrophone by the reciprocity

method in the vicinity of an air-water interface. Table I gives the results of this calculation for several separation-to-depth ratios. For most practical purposes a separation-to-depth ratio of 1/4 can be taken to give an unbiased calibration when the bottom surface is remote.

Table I. Bias in reciprocity calibration near an air-water interface with c-w projected energy; $b \equiv d/h$; $(p_H)_{aw}$ is pressure at the hydrophone near the interface; p_H is free-field pressure at the hydrophone; $B(M)$ is the relative bias; M is the free-field sensitivity; d is the distance from the center of a spherical radiator to the measuring point; and h is the distance from the radiator to the interface.

b	$\frac{(p_H)_{aw}}{p_H}$	$\frac{B(M)}{M}$	$B(M)$ (dB)
3.00	1.190	0.091	+0.76
2.00	1.133	0.064	+0.54
1.50	1.082	0.044	+0.37
1.00	1.051	0.025	+0.21
0.50	1.028	0.014	+0.12
0.25	1.004	0.002	+0.02

For peak detection under free-field conditions with negligible noise,

$$p_H \propto 1/d. \tag{39}$$

In the presence of an air-water interface, Eq. (33) applies for the pressure field at the hydrophone. Let $d_1 = (4h^2 + d^2)^{1/2}$. Then Eq. (33) can be written as

$$[p_H(d,t)]_{aw} \propto \frac{(d_1 \cos kd - d \cos kd_1) \sin \omega t + (d \sin kd_1 - d_1 \sin kd) \cos \omega t}{d d_1}. \tag{40}$$

The amplitude of the wave represented by Eq. (40) is the square root of the sum of the squares of the quadrature components, or

$$[p_H(d)]_{aw} \propto \frac{[d_1^2 + d^2 - 2dd_1 \cos k(d_1 - d)]^{\frac{1}{2}}}{dd_1}. \tag{41}$$

Averaging Eq. (41) over an interference cycle produces

$$(p_H)_{aw} \propto \frac{1}{2\pi} \int_0^{2\pi} \frac{[d_1^2 + d^2 - 2dd_1 \cos k\Delta x]^{\frac{1}{2}}}{dd_1} d(k\Delta x). \tag{42}$$

The ratio of the peak-detected pressures averaged over an interference cycle is then

$$\frac{(p_H)_{aw}}{p_H} = \frac{1}{2\pi} \left(\frac{4 + 2b^2}{4 + b^2}\right)^{\frac{1}{2}} \int_0^{2\pi} \left(1 - \frac{2b(4 + b^2)^{\frac{1}{2}}}{4 + 2b^2} \cos k\Delta x\right)^{\frac{1}{2}} d(k\Delta x). \tag{43}$$

Equations (37) and (43) are identical, which shows that the biases in reciprocity calibration for rms- and peak-detection near an air-water interface are the same when there is no noise. This result is as expected.

Statistical Concepts for the Analysis of Hydrophone Sensitivity

The proposition has been put forth that, under certain well specified conditions, reciprocity calibration leads to a set of estimates of hydrophone sensitivity that are unbiased and that are Gaussianly distributed. Here a review of the statistical parameters needed to specify the confidence coefficient associated with measurements from a Gaussian process will be given.

Associated with an infinite Gaussian population of estimates are two parameters necessary to specify the Gaussian probability density function for that population. These are the mean value M and the variance σ_M^2 defined by the equations

$$M = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \hat{M}_i \tag{44}$$

and

$$\sigma_M^2 = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} (\hat{M}_i - M)^2. \tag{45}$$

The Gaussian probability density function is given by

$$p(\hat{M}) = \frac{1}{\sqrt{2\pi}\,\sigma_M} \exp[-(\hat{M} - M)^2/2\sigma_M^2]. \tag{46}$$

It is well known that, when dealing with a finite number of estimates, the average of the estimates is a better estimate in a probabilistic sense than is a single estimate. The average value from a finite set of estimates taken from a potentially infinite set is defined by the equation

$$\bar{M} = \frac{1}{N} \sum_{i=1}^{N} \hat{M}_i, \tag{47}$$

and an unbiased estimate of the standard deviation is defined by the equation,

$$\hat{\sigma}_M = \left[\frac{1}{N - 1} \sum_{i=1}^{N} (\hat{M}_i - \bar{M})^2\right]^{1/2}. \tag{48}$$

The estimate for the average of N values of M is itself an unbiased Gaussian variable with a standard deviation (standard error)

$$\hat{\sigma}_{\bar{M}} = \hat{\sigma}_M/N^{1/2}. \tag{49}$$

The confidence interval about the mean associated with a single estimate is written as

$$(\hat{M} - A\sigma_M) < M < (\hat{M} + A\sigma_M), \tag{50}$$

where A is a constant to be determined. The confidence interval for the average of the estimates is written as

$$(\bar{M} - C\hat{\sigma}_M/N^{1/2}) < M < (\bar{M} + C\hat{\sigma}_M/N^{1/2}), \tag{51}$$

where C is a constant to be determined. The choice of the constants A and B depends upon the value of the confidence coefficient desired. There is a difficulty with the interval of Eq. (50) in that σ_M is not known for measurement of hydrophone sensitivity but can only be estimated by repeated measurements as $\hat{\sigma}_M$. Both $\bar{M}$ and $\hat{\sigma}/N^{1/2}$ in Eq. (51) are random variables. From Eq. (51), one defines

$$\left|\frac{\bar{M} - M}{\hat{\sigma}_M/N^{1/2}}\right| < C. \tag{52}$$

The random variable defined by the quantity inside of the absolute value signs is a random variable which, for a Gaussian $\bar{M}$, follows Student's t probability density functions, a family of functions that depend upon N. Like the Gaussian distribution, the Student t-distribution is symmetrical about the mean. As N becomes large, the Student t-distribution approaches the Gaussian distribution. The Student t-distribution can be used to compute the confidence interval associated with the average of the estimates and has been tabulated for various values of N and confidence coefficients. The confidence coefficient is defined as the area under the Student's t probability density function covered by the confidence interval. Because the true mean value is fixed, the confidence coefficient expresses the probability that the random confidence interval will bracket the mean.

What should one do if the estimates of hydrophone sensitivity do not follow a Gaussian distribution? This should cause no real alarm (as long as the bias in the measurement procedure is minimized) because of the central limit theorem. The central limit theorem is expressed as follows: Given a population of values with a finite variance, if we take independent samples from this population, all of size N, then the population formed by the averages of these samples will tend to have a Gaussian distribution, regardless of what the distribution is of the original population; the larger N, the greater will be this tendency toward the Gaussian distribution [1]. In practice N does not have to be very large for the distribution of the averages to be nearly Gaussian, so Eq. (51) will apply relatively well. Also, the Student t-distribution can be used to express the confidence coefficient. Because there are some differences in the Gaussian distribution and the distribution of the averages, high-confidence coefficients should not be used unless N is fairly large.

Confidence intervals can also be constructed for bias estimation. The bias estimate B is defined by

$$\hat{B} = \hat{M} - M. \tag{53}$$

The variance of $\hat{B}$ is the same as the variance of $\hat{M}$, since M is fixed, so the confidence interval for B is

$$(\bar{B} - C\hat{\sigma}_B/N^{\frac{1}{2}}) < B < (\bar{B} + C\hat{\sigma}_B/N^{\frac{1}{2}}), \tag{54}$$

where $\bar{B}$ is the average of the $\hat{B}$'s. The interval, like that for the confidence interval for the mean, can be found from tables of Student's t-distribution.

MEASUREMENTS AND ANALYSIS

In evaluating the accuracy and precision of existing calibration facilities at the USRD, the approach chosen was to calibrate two hydrophones by reciprocity at frequencies far below resonance using a minimum of equipment. These two hydrophones were calibrated with high accuracy

and precision several times, and the average values were chosen as the "true" values. Then the same hydrophones were calibrated three times at each calibration facility by normal procedures. It was recognized that three calibrations were too few to allow good evaluation, but workload requirements prevent making a large number of measurements at each facility. This section describes the measurements and gives an analysis of them.

Accurate and Precise Calibration of Two Standard Hydrophones

The two USRD standard hydrophones used in this study were the F37 serial A40 and the F50 serial 21. Drawings, photographs, and typical sensitivity curves for these two hydrophones are shown in the Appendix. These two hydrophones were chosen because both are cylindrical and they can be suspended by their cables without rigging to interfere with their calibration. Also, their characteristics are known to be relatively constant as a function of temperature and pressure. At 10 kHz and lower frequencies, both hydrophones can be considered omnidirectional in the horizontal plane because their diameters are small in comparison with a wavelength.

A decision was made to attempt to calibrate the standard hydrophones in such a manner as to ensure that a confidence interval of ±0.1 dB about an average value would contain the true mean with a confidence coefficient of 0.95 (95% certain to contain the true mean). Figure 6 is a block diagram of the system used in the reciprocity calibration of the standard hydrophones. With the Fluke voltmeter and the Pearson pulse current transformer, current can be read to within 0.01 dB. As the current transformer rating specifies a frequency response of 1 Hz to 35 MHz at

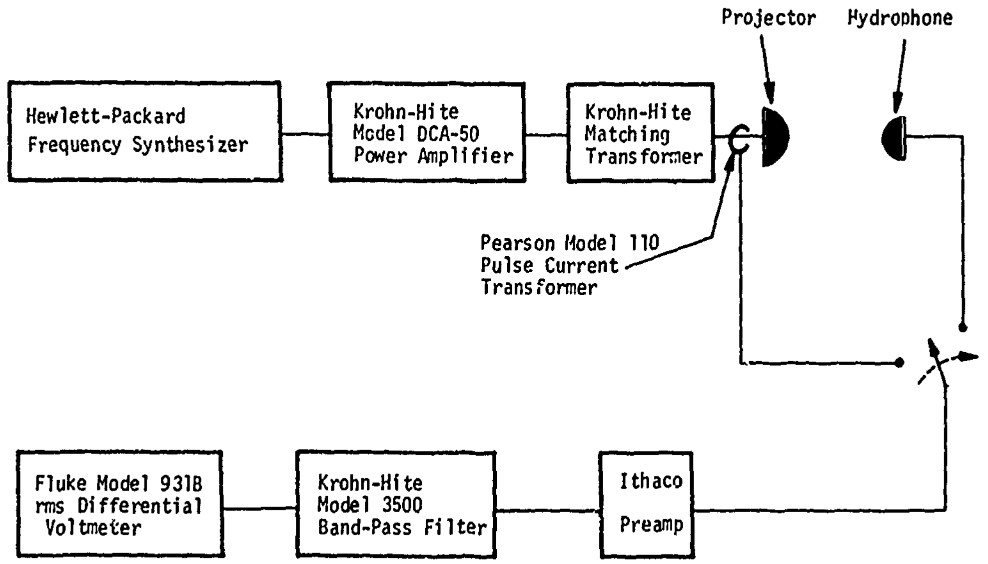


Fig. 6. Block diagram of simple calibration system.

the -3 dB points and an output voltage/ampere of 0.1 with a range of 0 to ±0.1 dB accuracy, the maximum hydrophone sensitivity bias expected from the current measurement bias is -0.05 dB, which can be deduced from Eq. (15). Measurement of open-circuit hydrophone voltages to within 0.1 dB is possible under some conditions. The environment in which open-circuit hydrophone output is measured determines how much this measured value will fluctuate. In Lake Gem Mary at the USRD, these fluctuations were so serious and the magnitude of the boundary interference so large that an unreasonably large number of calibrations would have been required to obtain the desired confidence interval with the simple system shown in Figure 6. Measurements made at the USRD's Leesburg Facility with the simple system and with the transducers at about mid-depth revealed that the hydrophone output voltage fluctuated less than 1% under most conditions. Exceptions occurred when the wind blew hard and when fish were in the vicinity of the transducers. Also, wider fluctuations occurred when the signal-to-noise ratio was insufficient and, at the lower frequencies, from interference by 60 Hz and its harmonics. Data were not recorded when fluctuations exceeded 2% of the voltmeter's reading and then only when the fluctuation was obviously due to wind conditions, which caused a very slow fluctuation, probably because of a lake seiche. One should recognize that the sound pressure field is changing under these conditions even though no measurable change in projector current may be observed, because the radiation impedance may be small in comparison with other impedances that limit the current.

The two hydrophones were calibrated sequentially in a single reciprocity calibration procedure with both the low-cutoff and high-cutoff frequencies of the Krohn-Hite filter set on the frequency at which the calibration was to be made. Frequencies chosen were 0.500, 1.000, 2.000, 5.000, and 10.000 kHz. Measurements were made (a) at the depth 21.06 m and separation 2.00 m with two F37's as the projector and reciprocal transducer, and (b) at the depth 17.36 m and separation 1.00 m with two J9's. The interference cycles for the air-water interface for these two set-ups are 37.3 and 44.5 Hz, respectively. Twelve frequencies in 3- and 4-Hz increments about the chosen frequencies were used to smooth out the interference cycle effects. The F37 and J9 transducers were checked for linearity and reciprocity. They were found to be both linear and reciprocal to within ±0.04 dB over the frequency and amplitude ranges in which they were used. A distance check was made to ascertain how accurately distance was being maintained. This was done by measuring 1- and 2-m separations on two free-hanging F37's and averaging the received hydrophone voltages over the 12 frequency values used in an interference cycle. The results of these measurements show that the distance is maintained to within ±1% of the measured distance of 1 m, which makes the distance error possibly the main source of error in these measurements. The error is larger with the F50 because it is lighter and therefore is more subject to cable deflections. Nevertheless, distance is essentially unbiased when it is measured repeatedly and averages are taken. Independence of the measurements was assured by making a reciprocity measurement at the 12 frequency points comprising one interference cycle. All measurements for that cycle (that is, three voltages and the current for each point in

the cycle) were made at the same instrument settings to minimize the effect of electronics on calibration.

Figures 7 and 8 show typical plots of the hydrophone sensitivities calculated by using the free-field reciprocity parameter with J9's used

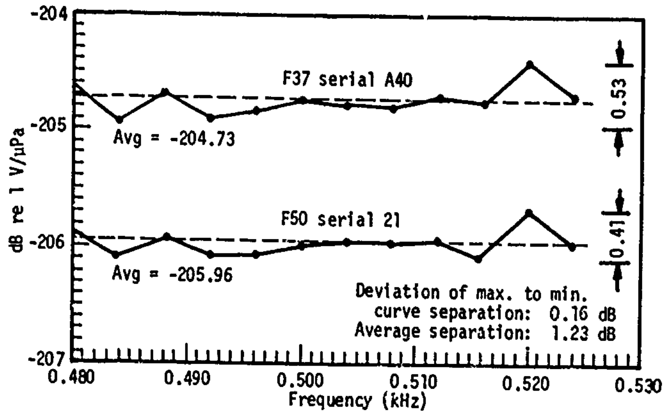
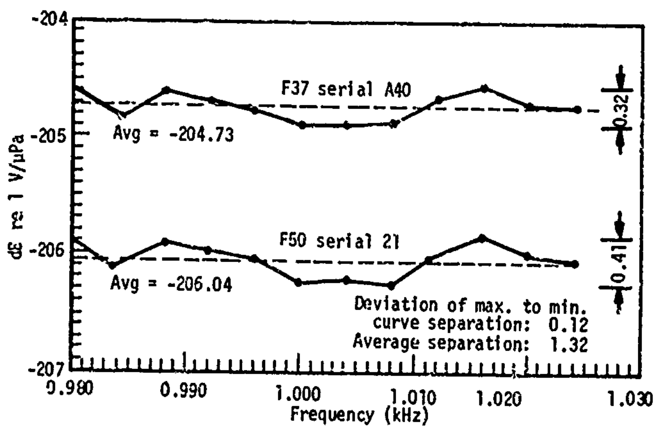


Fig. 7. Unsmoothed sensitivity plots for F37 and F50 hydrophones.



as projector and reciprocal transducer. These plots show the effects of boundary interference. It was calculated that the interference cycle due to the air-water interface alone was about 44 Hz and that the peak-to-peak sensitivity variation was about 0.3 dB. The maximum observed variation resulting from the interference cycle and measurement error combined for any calibration run was 0.8 dB; the average variation was about 0.5 dB. This fact, as well as the shapes of the plots in Figures 7 and 8, shows that boundaries other than the air-water interface affected the measurements. Averaging over the 12 points taken in the interference cycle is considered to provide an unbiased estimate of hydrophone sensitivity. Table II shows the results of sensitivity measurements made on hydrophones F37 serial A40 and F50 serial 21.

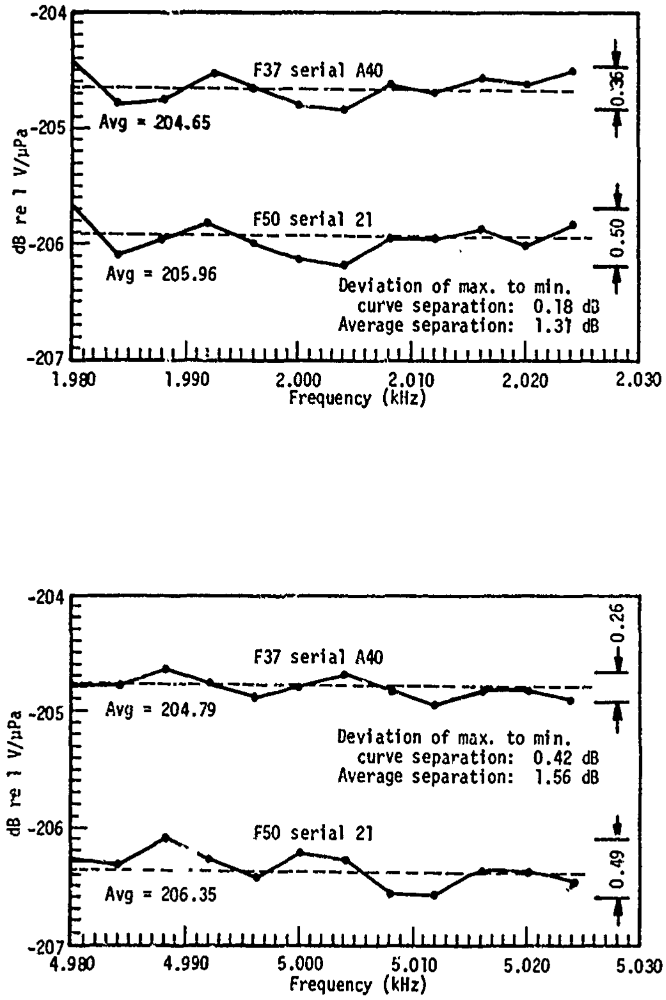


Fig. 8. Unsmoothed sensitivity plots for F37 and F50 hydrophones.

Table II shows that, with 15 independent estimates of the F37 hydrophone sensitivity averaged over 12 points of the interference cycle, the maximum spread of the data is 0.18 dB. The diffraction constant for the F37 should be nearly that for an infinite cylinder. Using the equation for the diffraction constant given by Henriquez [9], the effect of diffraction at 10 kHz is estimated to reduce the sensitivity by about 0.1 dB from that at lower frequencies. Taking the 12 estimates below 10 kHz as being estimates for the same true value, one obtains -204.71 dB re 1 V/μPa for their average. The standard deviation of these points is 0.062 dB, making the standard error of the average $\hat{\sigma}_{\bar{M}}$ to be approximately 0.02 dB. From Student's t-distribution table, the confidence interval for a confidence coefficient of 0.95 is $\bar{M} \pm 0.04$ dB. This means that the true value of sensitivity for the F37 serial A40 lies within the range -204.75 to -204.67 dB with 95% certainty if the measurement is unbiased as hypothesized. However, some small positive bias is sure to exist so that the wider range -204.80 to -204.67 dB with 95% certainty is not unreasonable.

Table II. Simple system calibration results; estimate of free-field voltage sensitivity $\hat{M}$ in dB re 1 V/μPa.

Hydrophone	Freq (kHz)	J9 Reciprocity		F37 Reciprocity		$\bar{M}$
F37	0.500	-204.73	-204.77			-204.7
	1.000	-204.73	-204.67			-204.7
	2.000	-204.65	-204.79	-204.62	-204.69	-204.7
	5.000	-204.79	-204.77	-204.62	-204.70	-204.7
	10.000	-204.59		-204.61	-204.67	-204.6
F50	0.500	-205.96	-205.96			-206.1
	1.000	-206.04	-206.09			-206.1
	2.000	-205.95	-206.03			-206.1
	5.000	-206.35	-206.10	-206.13	-206.34	-206.2
	10.000	-206.34		-206.18	-206.35	-206.3

The maximum spread of sensitivity data for the F50 hydrophone is 0.4 dB over the frequencies covered. Some of this spread is caused by the increased effect of diffraction of the F50 over that of the longer F37, and some is due to the increased distance error because the F50 weighs less and has a lighter cable than the F37, which reduces its tendency to hang straight when freely suspended. The diffraction effect is estimated to reduce sensitivity at 10 kHz by from 0.15 to 0.20 dB below that at lower frequencies. At 500 Hz the signal-to-noise ratio was so low that the estimates were judged to be biased by about +0.1 dB. Taking the 10 estimates below 10 kHz as being estimates for the same true value, one obtains -206.11 dB re 1 V/μPa for their average. The standard deviation of these points is 0.13 dB, making the standard error of the average to be about 0.04 dB. Tables for Student's t-distribution indicate that the confidence interval is -206.21 to -206.01 dB with a confidence coefficient of 0.95. Again, because of the possibility of positive bias, the interval -206.26 to -206.01 dB with 95% certainty is not unreasonable.

In view of the planned design of a new calibration system for the Leesburg Facility, it seems appropriate to say what probability interval may be expected from reciprocity measurements based on the methods employed here. One would expect a probability interval of about ±0.30 dB to have a probability of 0.95 of containing the mean value of hydrophone sensitivity for the F50 with comparable values of about ±0.15 dB and 0.95 for the F37. These intervals can be reduced by closer attention to signal-to-noise ratios, environmental conditions, and distance measurements. By using the sing-around velocimeter principle to measure distance at the depth of the suspended transducers instead of at the top of the rigging, it may be possible to achieve smaller probability intervals.

A Preliminary Study of the Precision and Accuracy of Hydrophone Calibration at the Established USRD Facilities

In this section, the measurements of hydrophone sensitivity made at the Leesburg Facility, the Anechoic Tank Facility, System F Tube Facility, Center Pier Facility, Digital Measuring System Facility, and North Pier Facility are presented. To whatever extent is possible, statistical statements about errors are based on a very limited amount of data and on the use of the average values from the calibrations of the F37 and the F50 as "true" values. Also, estimates at the different frequencies were assumed independent for a single run, even though the transducers were not rerigged.

Leesburg Calibration Facility

Three calibrations each were made on the F37 serial A40 and the F50 serial 21 at the depth 1480 cm and separation 100 cm. These were comparison calibrations with the F37 serial 5 (previously calibrated by reciprocity) used as the standard. Table III shows the results of these calibrations and their comparisons with the "true" values. In this table, B denotes the difference between estimated sensitivity and "true" sensitivity.

Table III. Free-field voltage sensitivity $\hat{M}$ (dB re 1 V/μPa) and bias estimate $\hat{B}$ for two hydrophones measured at the Leesburg Facility.

Hydrophones	Freq (kHz)	$\hat{M}_1$	$\hat{B}_1$	$\hat{M}_2$	$\hat{B}_2$	$\hat{M}_3$	$\hat{B}_3$
F37	0.5	-204.3	+0.4	-204.5	+0.2	-204.5	+0.2
	1.0	-204.3	+0.4	-204.6	+0.1	-204.5	+0.2
	2.0	-204.3	+0.4	-204.6	+0.1	-204.4	+0.3
	5.0	-204.3	+0.4	-204.5	+0.2	-204.4	+0.3
	10.0	-204.4	+0.2	-204.7	-0.1	-204.6	0
F50	0.5	-205.8	+0.3	-205.8	+0.3	-205.9	+0.2
	1.0	-205.8	+0.3	-205.9	+0.2	-205.9	+0.2
	2.0	-205.7	+0.4	-205.9	+0.2	-205.9	+0.2
	5.0	-205.8	+0.3	-206.0	+0.1	-206.0	+0.2
	10.0	-206.0	+0.3	-206.1	+0.1	-206.2	+0.1

The maximum deviation of these results from the "true" value is +0.4 dB. The estimated bias $\hat{B}$ for the F37 serial A40 is +0.22 dB. If one assumes distance error is negligible, then there are 15 independent estimates on each hydrophone. Maximum deviation on repeatability is 0.3 dB. The estimated standard error, about B for the F37, is $\hat{\sigma}_{\bar{B}}$ = 0.05 dB, which, using the Student t-distribution, gives the confidence interval for the bias as approximately +0.12 < B < +0.32 dB with 95% certainty. This amount of bias can be due to a bias in the calibration of the F37 serial 5 used as a standard. If one assumes this to be the case and if comparison

were made using an unbiased standard, confidence limits at the Leesburg Facility for calibrating an F37 would be about ±0.4 dB with 95% certainty, assuming a normal distribution and that $\hat{\sigma}_M \simeq \sigma_M$.

The estimated bias for the F50 serial 21 is +0.23 dB. Again assuming negligible distance error, the estimated standard error about $\bar{B}$ is 0.01 dB which, using Student's t-distribution, gives the confidence interval for the bias as $0.21 < B < 0.25$ dB with 95% certainty. If the biases were due to calibration of the F37 serial 5 alone, then comparison with an unbiased standard would give confidence limits of about ±0.1 with 95% certainty.

In calibrating both the F37 serial A40 and the F50 serial 21, the estimated bias was about 0.2 dB. The source of the bias was not ascertained, but would seem to be either a distance bias or bias in the sensitivity of the standard used in the comparison.

Anechoic Tank Calibration Facility

Each of the three calibrations made by the Anechoic Tank Facility on the two test hydrophones are shown in Table IV. These measurements were made by comparison with a "known" standard.

Table IV. Free-field voltage sensitivity $\hat{M}$ (dB re 1 V/μPa) and bias estimate $\hat{B}$ for two hydrophones measured in the Anechoic Tank Facility

Hydro-phones	Freq (kHz)	$\hat{M}_1$	$\hat{B}_1$	$\hat{M}_2$	$\hat{B}_2$	$\hat{M}_3$	$\hat{B}_3$
F37	0.5	-204.8	-0.1	-204.5	+0.2	-204.5	+0.2
	1.0	-204.8	-0.1	-204.5	+0.2	-204.5	+0.2
	2.0	-204.8	-0.1	-204.5	+0.2	-204.5	+0.2
	5.0	-204.8	-0.1	-204.5	+0.2	-204.5	+0.2
	10.0	-204.8	-0.2	-204.5	+0.3	-204.5	+0.3
F50	0.5	-206.2	-0.1	-206.0	+0.1	-206.0	+0.1
	1.0	-206.1	0	-206.0	+0.1	-206.0	+0.1
	2.0	-206.1	0	-206.0	+0.1	-206.0	+0.1
	5.0	-206.2	0	-206.0	+0.2	-206.0	+0.2
	10.0	-206.2	+0.1	-206.0	+0.3	-206.0	+0.3

The maximum deviation from "true" value on these measurements is +0.3 dB. Estimated biases for the F37 and F50 are +0.08 and +0.11 dB, respectively. Maximum deviation from repeatability is 0.5 dB, while the estimated standard errors about the $\bar{B}$'s are 0.05 dB for the F37 and 0.03 dB for the F50. These values correspond to confidence intervals of $-0.03 < B < +0.19$ dB and $+0.05 < B < +0.17$ dB, respectively, at the 95% confidence level. The F37 cannot be said to have a bias in its estimated sensitivity, but the F50 can at the 95% level of certainty. Confidence intervals on estimated hydrophone sensitivities calibrated in the Anechoic

Tank are about $\hat{M} - 0.1 < M < \hat{M} + 0.3$ dB for the F37 and $\hat{M} + 0.0 < M < \hat{M} + 0.2$ dB for the F50 to about 95% certainty.

Low Frequency Facility, System K

Measurements made in System K on the two standard hydrophones were limited to frequencies at or below 1 kHz. Comparison is made to reference values of -204.7 dB re 1 V/ Pa for the F37 and -206.1 dB re 1 V/μPa for the F50. Results of these measurements are shown in Table V. These measurements were made by comparison with a small hard hydrophone that previously had been calibrated in a coupler.

Table V. Free-field voltage sensitivity $\hat{M}$ (dB re 1μV/ Pa) and bias estimate $\hat{B}$ for two hydrophones measured in the Low Frequency Facility, System K.

Hydrophones	Freq (Hz)	$\hat{M}_1$	$\hat{B}_1$	$\hat{M}_2$	$\hat{B}_2$	$\hat{M}_3$	$\hat{B}_3$
F37	10	-204.7	0	-204.8	-0.1	-204.7	0
	20	-204.7	0	-204.8	-0.1	-204.7	0
	50	-204.7	0	-204.8	-0.1	-204.7	0
	100	-204.7	0	-204.8	-0.1	-204.7	0
	200	-204.7	0	-204.8	-0.1	-204.7	0
	500	-204.6	+0.1	-204.8	-0.1	-204.6	+0.1
	800	-204.6	+0.1	-204.8	-0.1	-204.6	+0.1
	1000	-204.6	+0.1	-204.8	-0.1	-204.6	+0.1
F50	10	-205.8	+0.3	-205.8	+0.3	-205.8	+0.3
	20	-205.8	+0.3	-205.8	+0.3	-205.8	+0.3
	50	-205.8	+0.3	-205.8	+0.3	-205.8	+0.3
	100	-205.8	+0.3	-205.8	+0.3	-205.9	+0.2
	200	-205.9	+0.2	-205.8	+0.3	-205.9	+0.2
	500	-205.8	+0.3	-205.8	+0.3	-206.0	+0.1
	800	-205.9	+0.2	-205.8	+0.3	-206.1	0
	1000	-205.9	+0.2	-205.8	+0.3	-205.2	-0.1

The maximum deviations from "true" values on the F37 and F50 are ±0.1 dB and +0.3 dB, respectively. Estimated bias $\hat{B}$ is -0.01 dB for the F37 and +0.24 for the F50. Maximum deviation from repeatability is 0.3 dB, while the estimated standard errors about the $\bar{B}$'s are 0.03 dB for the F37 and 0.04 for the F50. These values correspond to confidence intervals of $-0.02 < B < 0.00$ for the F37 and $+0.22 < B < +0.26$ for the F50 with 95% certainty. Confidence intervals on the estimated hydrophone sensitivities calibrated with System K are about $\hat{M} - 0.1 < M < \hat{M} + 0.1$ dB for the F37 and about $\hat{M} + 0.2 < M < \hat{M} + 0.3$ dB for the F50 with about 95% certainty.

Lake Gem Mary Facility, Center Pier System

Results of measurements made by comparison for the two standard hydrophones are shown in Table VI.

Table VI. Free-field voltage sensitivity $\hat{M}$ (dB re 1 V/μPa) and bias estimate $\hat{B}$ for two hydrophones measured in the Lake Gem Mary Facility, Center Pier System.

Hydrophone	Freq (kHz)	$\hat{M}_1$	$\hat{B}_1$	$\hat{M}_2$	$\hat{B}_2$	$\hat{M}_3$	$\hat{B}_3$
F37	0.5	-205.0	-0.3	-204.5	+0.2	-205.2	-0.5
	1.0	-205.0	-0.3	-204.5	+0.2	-205.2	-0.5
	2.0	-204.9	-0.2	-204.6	+0.1	-204.9	-0.2
	5.0	-205.0	-0.3	-204.5	+0.2	-205.0	-0.3
	10.0	-204.6	0	-204.5	+0.1	-204.8	-0.2
F50	0.5	-206.0	+0.1	-206.0	+0.1	-206.4	-0.3
	1.0	-206.0	+0.1	-206.0	+0.1	-206.4	-0.3
	2.0	-206.1	0	-206.0	+0.1	-206.3	-0.2
	5.0	-206.3	-0.1	-206.3	-0.1	-206.0	+0.2
	10.0	-206.2	+0.1	-206.3	0	-206.2	+0.1

The maximum deviations from "true" values on the F37 and F50 are -0.5 dB and -0.3 dB, respectively. Estimated bias $\hat{B}$ is -0.13 for the F37 and -0.07 for the F50. Maximum deviation from repeatability is 0.7 dB, while the estimated standard errors about the $\bar{B}$'s are 0.06 dB for the F37 and 0.05 dB for the F50. These values correspond to confidence intervals of $-0.25 < B < -0.01$ dB and $-0.18 < B < +0.04$ dB for the F37 and F50, respectively, with 95% certainty. Confidence intervals on the estimated hydrophone sensitivities calibrated with the center pier system are about $\hat{M} - 0.4 < M < \hat{M} + 0.1$ dB for the F37 and about $\hat{M} - 0.3 < M < \hat{M} + 0.1$ dB for the F50 with about 95% certainty.

Lake Gem Mary Facility, Digital Measuring System

Results of measurements made by comparison for the two standard hydrophones are shown in Table VII.

The maximum deviation from "true" values on the F37 and F50 are -1.0 dB and -0.6 dB respectively. Estimated bias $\hat{B}$ is -0.72 dB and -0.19 dB for the F37 and F50, respectively. Maximum deviation from repeatability is 0.6 dB, while the estimated standard errors about the $\bar{B}$'s are 0.03 dB for the F37 and 0.06 dB for the F50. These values correspond to confidence intervals of $-0.78 < B < -0.66$ and $-0.31 < B < -0.07$ dB for the F37 and F50, respectively, with 95% certainty. Confidence intervals on the estimated hydrophone sensitivities calibrated with the digital measuring system are about $\hat{M} - 1.0 < M < \hat{M} - 0.5$ dB for the F37 and about $\hat{M} - 0.7 < M < \hat{M} + 0.2$ dB for the F50 with about 95% certainty.

Table VII. Free-field voltage sensitivity $\hat{M}$ (dB re 1 V/μPa) and bias estimate $\hat{B}$ for two hydrophones measured in the Lake Gem Mary Facility, Digital Measuring System.

Hydrophone	Freq (kHz)	$\hat{M}_1$	$\hat{B}_1$	$\hat{M}_2$	$\hat{B}_2$	$\hat{M}_3$	$\hat{B}_3$
F37	0.5	-205.4	-0.7	-205.3	-0.6	-205.5	-0.8
	1.0	-205.5	-0.8	-205.5	-0.8	-205.5	-0.8
	2.0	-205.4	-0.7	-205.3	-0.6	-205.7	-1.0
	5.0	-205.5	-0.8	-205.3	-0.6	-204.7	-0.6
	10.0	-205.2	-0.6	-205.4	-0.8	-204.6	-0.6
F50	0.5	-206.3	-0.2	-206.4	-0.3	-206.3	-0.2
	1.0	-206.1	-0.1	-206.5	-0.4	-206.3	-0.2
	2.0	-206.2	-0.1	-206.4	-0.3	-206.3	-0.2
	5.0	-206.4	-0.2	-206.3	-0.1	-205.8	+0.4
	10.0	-206.7	-0.4	-206.9	-0.6	-206.3	0

Lake Gem Mary Facility, North Pier System

Measurements made on the North Pier System showed that the system was not suitable for calibrating at that time. No calibrations were being performed on the system at that time because facility personnel felt the system was inaccurate. Measurements showed a -1.0 to -1.5 dB bias in calibrating the F37 serial A40 and F50 serial 21 with some values as high as -1.8 dB off. The system has been repaired and is said to be working properly, but no new measurements have been made on it for this report.

SUMMARY AND DISCUSSION OF SYSTEM ERROR MEASUREMENTS

Figure 9 is a precision and accuracy diagram for all of the systems for both the F37 serial A40 and the F50 serial 21 measurements relative to the estimates obtained with the simple calibration system as the "true" value. The estimated bias $|\bar{B}| = |\bar{M} - \text{"M"}|$ is shown for each hydrophone as calculated from measurements made on each system. Also shown are the confidence intervals associated with the "true" value "M". The upper of the pair of confidence intervals represents the 95% certainty for $\bar{M}$. The lower confidence interval is for $\hat{M}$ of the probability that 95% of the estimates for hydrophone sensitivity made from single measurements will fall within the marked interval, assuming $\hat{\sigma}_M \simeq \sigma_M$ and a normal distribution for $\bar{M}$. The X's indicate values that fell outside of the confidence interval, while the O's indicate other extreme departures from $\bar{M}$. Out of 150 values, only 6 fell outside the confidence intervals (about 96% were within the intervals).

From these measurements, one can conjecture that the USRD is very well justified in claiming that an interval of ±1 dB about the true value of hydrophone sensitivity will contain at least 95% of the estimates of

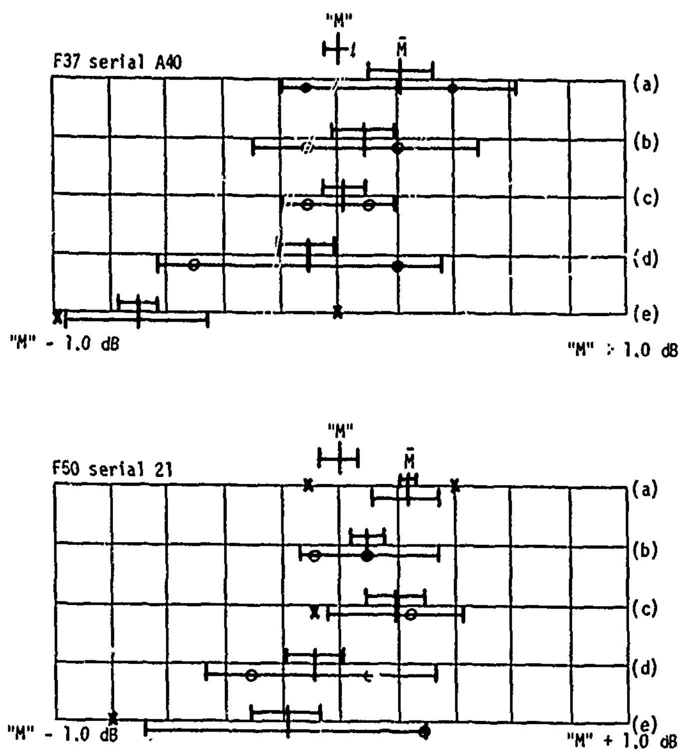


Fig. 9. Precision and accuracy diagram for hydrophone sensitivity measurements at USRD facilities: (a) Leesburg, (b) Anechoic Tank, (c) System K, (d) Center Pier, (e) Digital Measuring System.

hydrophone sensitivity for small standard hydrophones operated far below resonance. At 90% certainty all systems except the digital measuring system seem to have an interval of ±0.5 dB associated with them. Carefull attention to sources of bias, repetitive calibration, and modern signal processing techniques probably could lead to a confidence interval of ±0.1 to ±0.2 dB without much added system complexity.

RECOMMENDATIONS

A. Study of system accuracy should continue with a system-by-system approach instead of using an over-all approach in order to obtain enough data from a system to be able to pinpoint random and systematic errors.

B. A study of distance errors associated with rigging of transducers should be undertaken. This could be done with sufficient accuracy by application of the sing-around velocimeter principle.

C. A study of signal processing techniques aimed at calibration with low signal-to-noise ratios is recommended. This could lead to better free-field calibration at low frequencies where noise is high and where difficulty is encountered in building small, high-intensity sound sources.

D. A study of the radiation impedance of standard hydrophones is recommended with a view toward developing cutoff filters for some standards, which would limit their use to the frequency range where they can be calibrated and used without too much attention to boundary conditions.

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The author wishes to acknowledge the outstanding cooperation and assistance of USRD supervisors and staff, particularly that of Virgil Apostolico, who helped to obtain data for the hydrophones used as "true" value standards; and facility heads James U. Walker, Harris J. Hebert, Charles R. Bobo, and Vincent P. Benedetti, who provided information required to evaluate facility errors. Also acknowledged are the generous contributions of time by supervisors and other staff for intelligent discussions to promote a better understanding of hydrophone calibration and associated problems.

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APPENDIX A

This appendix consists of photographs, drawings, and typical sensitivity curves for the F37 and F50 hydrophones.

Fig. A1. F37 hydrophone.

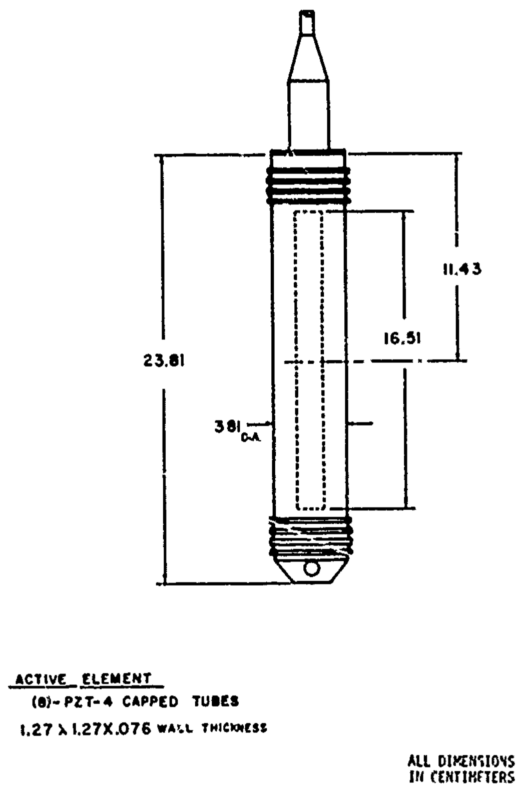

Fig. A2. Nominal dimensions of F37 hydrophone.

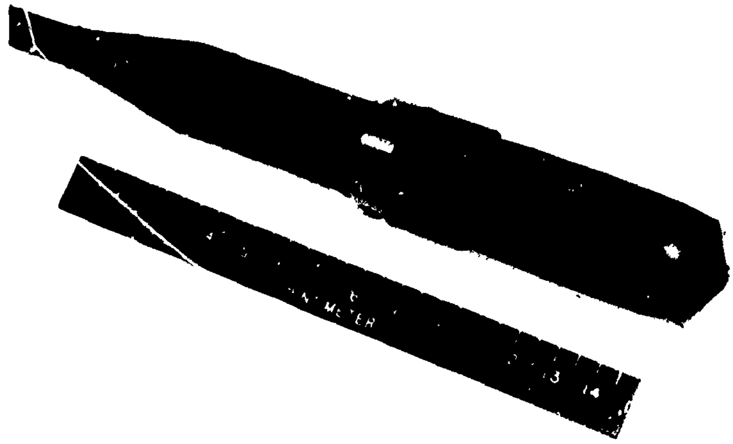

Fig. A3. F50 hydrophone.

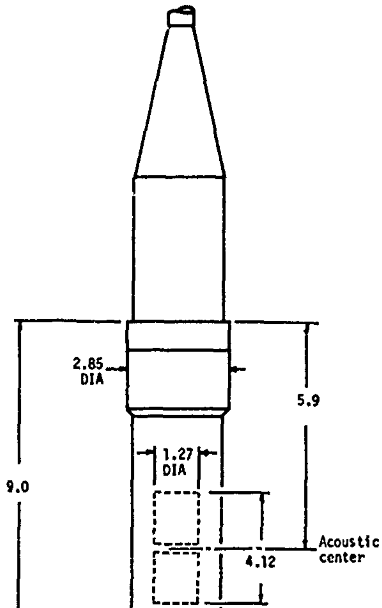

Fig. A4. Nominal dimensions of F50 hydrophone.

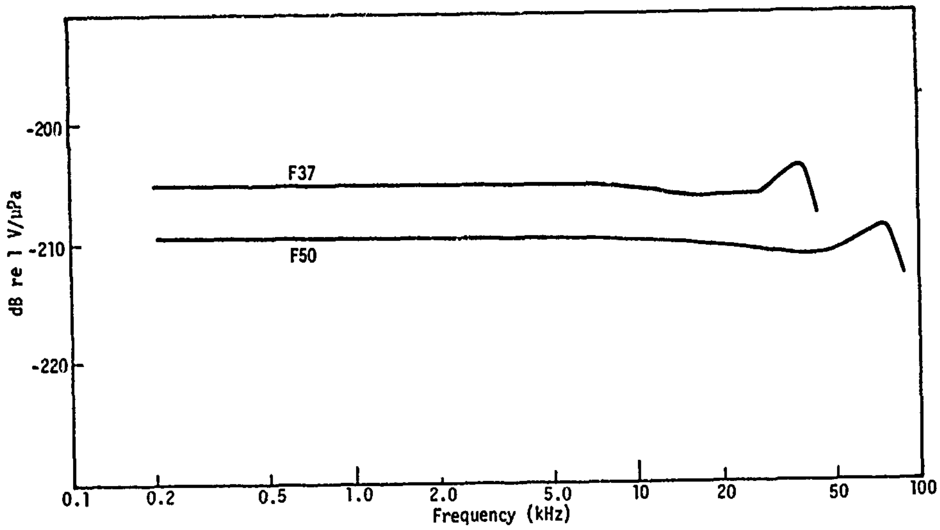

Fig. A5. Typical free-field voltage sensitivity curves for F37 and F50 hydrophones.